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A Compatibility Assessment Method for Adaptive-Wall Wind Tunnels

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A new method is described for assessing the compatibility between inner and outer flow regimes in adaptive-wall wind tunnels. The method is applicable to both two- and three-dimensional flows and, unlike other schemes, requires the measurement of only one velocity component. Moreover, a complete solution to the outer flowfield is not required with the new method. Computer simulations of two- and three-dimensional flows are presented along with data from a two-dimensional pilot wind tunnel test using the new method.

I. Introduction

A MAJOR uncertainty in the interpretation of transonic wind tunnel data is the effect of the walls on the measured data. Over many years empirical rules were developed regarding acceptable model blockage ratios for minimizing wall interference.[1] These rules usually dictate that the total model cross section be no greater than about 1% of the test section cross-sectional area. Strict adherence to these rules usually requires that wind tunnels with large test sections or high stagnation pressures be used to obtain Reynolds numbers approaching those found in flight.

To overcome these limitations, many attempts to develop analytical models of the flow in transonic test sections have been made (see Ref. 2 for a comprehensive summary), but no simple rational method was developed that is useful over a wide range of conditions. As the size and speed of digital computers have increased, the main effort to model transonic test-section flow has switched to the development of numerical methods. In the initial attempts at numerical modeling,[3-5] boundary conditions were imposed at the walls themselves, but realistic physical modeling is difficult due to the complex nature of the flow through the ventilated (usually holes or slots) test-section walls. Recent studies[6-9] have bypassed the difficult question of wall modeling by imposing measured flow quantities (pressures or velocities) on a suitably chosen control surface as boundary conditions.

Over the past few years a new approach to the problem of wind tunnel wall interference has been evolving. The method is known as the adaptive-wall technique and is based on the premise that if a streamline above and below the model (a streamtube in three-dimensional flow) could be deformed into its interference-free shape, then the forces and pressures on the model would represent free-air data. Aside from some early British work on streamlined walls,[10] the first modern treatments of the adaptive-wall concept were published by Ferri and Baronti[11] and by Sears.[12]

The work of Ferri and Baronti was aimed at developing a method for assessing wall interference through the measurement of two independent flow quantities near the tunnel boundaries. The flow variables chosen were flow deflection and wall static pressure. The contribution of Ferri and Baronti to adaptive-wall technology was their pointing out of the necessity of performing a free-air computation in the far field and comparing the results with measured data. In addition, they discussed the validity of using linearized theory for transonic wall interference assessment and demonstrated its applicability by numerical computations.

In a simultaneous and independent work, Sears developed the basic idea of the modern adaptive-wall wind tunnel. He developed the concept of a control surface removed from the walls where computed and measured flow properties are compared to determine their consistency with an unconfined flow. In Sears' method, one of the flow parameters is used as a boundary condition to compute an unconfined flow external to the chosen control surface. This calculation yields a prediction of the other flow property, which is in turn compared with the measured value to assess compatibility. The wall conditions (variable ventilation or geometry) are changed iteratively until the computed and measured values agree, implying interference-free flow in the wind tunnel. The strongest feature of this approach is that the difficulty of modeling or calibrating the flow near the wall is avoided.

Sears' approach was studied experimentally to assess its applicability to two-dimensional flow,[13] and the concept was extended to three-dimensional flows in a feasibility study by Erickson.[14] This method works satisfactorily for two-dimensional flows, but has some drawbacks for three-dimensional applications, namely in the difficulty of determining a distribution of singularities that will describe adequately a complex three-dimensional flow.

Other compatibility assessment procedures were also developed for use with adaptive-wall wind tunnels.[15,16] For the self-streamlining wind tunnel (deforming walls), calculation procedures based on slopes and pressures (similar to those proposed by Ferri and Baronti) were used. In some exploratory three-dimensional studies[17] a method based on a complete solution to the transonic small-disturbance equations was developed. Each of these methods requires a complete calculation of the external flowfield.

If a successful adaptive-wall wind tunnel is to be built, the compatibility assessment procedure must be developed with due regard to the sensor system used to measure the flow. Moreover, the method must be simple, reliable, and amenable to rapid calculation using on-line computer facilities. The new method described in this paper was developed specifically to exploit the capabilities of advanced laser systems for the measurement of fluid flow. The simplest such instrument—a one-component laser anemometer—is the only sensor needed for wall compatibility assessment using the new method. The procedure is valid for both two- and three-dimensional flows and was developed especially for use with a minicomputer-equipped facility. The method differs from other proposed methods in two significant ways: 1) only one velocity component is measured on two surfaces, rather than two components on one surface; and 2) a complete solution for all three velocity components are not required for compatibility assessment.

In Sec. II, the equations governing the flowfield in the outer region are developed and the method is described. In Sec. III, a simple formula is derived for compatibility assessment in

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two dimensions. The formula is applied to a sample problem and is validated with data from a recently completed pilot adaptive-wall wind tunnel test.[18] In Sec. IV, the procedure is extended to three-dimensional flows and a computational algorithm is described for three-dimensional applications. The analysis in this paper is restricted to linearized subsonic flows. This restricts the domain of transonic flow, if it exists, to lie entirely within the innermost control surface. Research is under way to extend the ideas to fully transonic flows and to determine the practical limits of the linearized analysis.

II. Equations of Motion and Outline of the Method

In most textbook derivations, the equations of motion for an ideal gas that neglect all dissipative and nonlinear effects are prescribed in terms of the perturbation velocity potential.[19] The velocity potential formulation is a natural choice since the entire flowfield can be computed easily from its derivatives. A slight complication exists in that the physical boundary conditions are given in terms of velocities; this means that a second boundary value problem (Neumann problem) needs to be solved. This usually involves more sophisticated coding in numerical implementations. A complete solution is not actually necessary for the purposes of compatibility assessment. All that is needed is a method for determining when a portion of the flow—the flow beyond a certain control surface—is unconfined.

The equations governing the perturbation potential to a steady, nondissipative, uniform flow at Mach number M is

$$(1-M^2)\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=0 \qquad (1)$$

where $u=\partial\phi/\partial x$, $v=\partial\phi/\partial y$, and $w=\partial\phi/\partial z$ are the perturbation velocities in the streamwise (x), lateral (y), and vertical (z) directions, respectively. By differentiating Eq. (1) with respect to z, a similar equation for the upwash can be derived,

$$(1-M^2)\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}=0 \qquad (2)$$

An application of the well-known Prandtl-Glauert transformation converts Eq. (2) into the familiar Laplace equation in the case $M<1$. The advantage of formulating the interference assessment problem in terms of the upwash will be discussed subsequently.

The compatibility assessment method consists of four steps: 1) measuring the boundary conditions for Eq. (2) at a suitably chosen control surface; 2) solving the boundary value problem for w; 3) computing w at a second control surface; and 4) comparing the results of step 3 with measurements at the second control surface. In this paper it will be assumed that the Mach number is less than unity (elliptic partial differential equation), and Eq. (2) will be referred to as the upwash equation. In the first step, a control surface is chosen in the flow. On this surface the upwash is measured at a number of control points. (The number and distribution of control points, especially for three-dimensional flows, constitutes the "art" of adaptive-wall research.) These measurements, along with uniform flow conditions at infinity, define a well-posed boundary value problem for the upwash—the first exterior boundary value problem (Dirichlet problem) for Laplace's equation. Depending on the number of space dimensions, either an analytical or numerical solution can be readily computed. Once a solution to the upwash equation is obtained, the upwash on the second surface is computed according to step 3. In the last step the computed upwash on the second surface is compared with measured values, any discrepancies being a measure of wind-tunnel wall interference. (Complete coincidence would indicate an interference-free flow.) The data at the second control surface are then coupled with a suitable control law for resetting the walls of the adaptive-wall wind tunnel. The corrected flow is then remeasured and the compatibility assessment procedure repeated. The first practical implementation of the method was described in the aforementioned experimental study.[18]

There are several significant differences between this method and others currently in use. First, a solution for the complete perturbation flowfield is not needed, although it could be found if desired. Second, a Dirichlet problem rather than a Neumann problem must be solved. (Although no real conceptual differences exist, numerical implementation of the Neumann problem in three dimensions requires some extra calculations.) Most important, however, is the simplification to instrumentation and sensor requirements. In a three-dimensional problem, the full solution to Eq. (1) would require that the velocity component normal to the control surface be measured. If a rectangular parallelepiped is the chosen surface, normal velocities in the cross-flow plane would include both upwash and sidewash, and instrumentation to measure at least two velocity components is needed. Using the two-surface method and the solution of Eq. (2) only one component of velocity ever needs to be measured. This probably will result in a significant saving in instrumentation complexity, expense, and data reduction software. For transonic flows it may not be possible to construct an upwash equation that is independent of other velocity components; however, it may be feasible to utilize another velocity component or the pressure.

III. Two-Dimensional Flow

A two-dimensional compatibility assessment problem is posed in Fig. 1, where an airfoil is shown immersed in a subsonic flow. The freestream Mach number, however, can be sufficiently large to induce a region of supersonic flow in the vicinity of the body. Four control lines are considered, two above and two below the test airfoil. The inner lines (source level) separate the flow into three regions, a semi-infinite region above the airfoil, a finite strip that includes the model, and a semi-infinite region below the airfoil. As will be shown presently, the first and third regions are effectively decoupled since boundary conditions are imposed independently on the boundaries of regions I and III. This vastly simplifies the problem since only one analysis suffices for either the symmetric thickness problem, the antisymmetric lifting flat-plate problem, or the practical case of lifting airfoil. In fact, the entire analysis turns out to be equivalent to the classical "thickness problem" of thin-airfoil theory. This is perhaps the simplest problem in potential theory because the solution is expressed easily as an integral over a distribution of elementary singularities.

With the usual assumptions that the flow at the control surface (and beyond) satisfies the conditions of linearized small perturbation theory, the mathematical problem in regions I and III (Fig. 1) is the solution to a classical Dirichlet

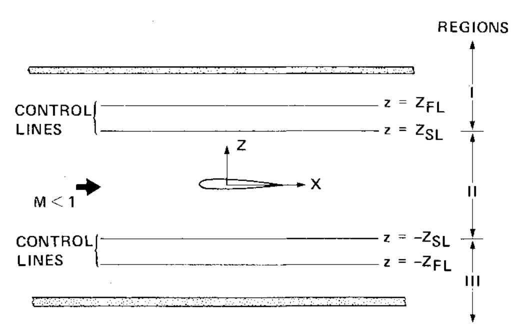

Fig. 1 Interference assessment in two dimensions.

problem for the two-dimensional form of Eq. (2),

$$\beta^2 \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} = 0 \tag{3}$$

where $\beta^2 = 1 - M^2$. The differential equation is subject to the following boundary conditions: In region I ($z \geq Z_{SL}$, $-\infty < x < \infty$),

$$w = W_{SL}(x) \text{ on } z = Z_{SL}, \quad -\infty < x < \infty$$

and

$$w \to 0 \text{ for } x^2 + z^2 \to \infty$$

In region III ($z \leq -Z_{SL}$, $-\infty < x < \infty$),

$$w = W_{SL}(x) \text{ on } z = -Z_{SL}, \quad -\infty < x < \infty$$

and

$$w \to 0 \text{ for } x^2 + z^2 \to \infty$$

where $W_{SL}(x)$ refers to the given (measured) upwash at the levels $z = \pm Z_{SL}$, respectively.

Each of these classical boundary-value problems has a simple solution. The solutions are given by

$$w(x,z) = \frac{\beta \, | z - Z_{SL} |}{\pi} \int_{-\infty}^{\infty} \frac{W_{SL}(x')\,dx'}{(x-x')^2 + \beta^2 (z - Z_{SL})^2} \tag{4}$$

in both regions I and III. The upwash in either region depends only on the bounding upwash in that region. This effectively decouples the flow in regions I and III for the purpose of compatibility assessment. This decoupling is strictly applicable for infinitely long control surfaces, and in practice measurements must be made far enough away from the model to insure that contributions from the "tails" of Eq. (4) are small. Note that three-dimensional problems cannot be decoupled in this manner since the trailing vortex system induces a finite upwash in the far wake.

The upwash at the level of compatibility assessment is

$$W_{FL}(x) = \frac{\beta \, | Z_{FL} - Z_{SL} |}{\pi} \int_{-\infty}^{\infty} \frac{W_{SL}(x')\,dx'}{(x-x')^2 + \beta^2 (Z_{FL} - Z_{SL})^2} \tag{5}$$

The previously defined four steps of the method are now quite clear: 1) measure $W_{SL}(x)$ at the two levels $z = \pm Z_{SL}$; 2) and 3) compute the unconfined upwash $W_{FL}(x)$ at $z = \pm Z_{FL}$ using Eq. (5); and 4) compare with the measured upwash at the field level. Since the solution given by Eq. (5) is in integral form, and integration is fundamentally a smoothing operation, small experimental errors in $W_{SL}(x)$ will not seriously compromise the solution. Moreover, the kernel of the integrand is always positive and nonsingular using the two-surface method. Very simple numerical integration algorithms can be used to process the upwash data.

The first check on the procedure was to determine the validity of Eq. (5) by using an analytically defined flow as input. The flow selected was the linearized subsonic velocity field about an NACA 0012 airfoil at $M = 0.65$, $\alpha = 2$ deg. The solution is obtained easily as a superposition of the basic thickness and lifting flat-plate problems. The approach used was to compute the flow for free-air conditions so that Eq. (5) would reproduce free-air velocities at Z_{FL}. The analytical solution for the upwash is shown in Fig. 2 at the selected levels $Z_{SL} = \pm C$ and $Z_{FL} = \pm 1.67\ C$, using a solid line at Z_{SL} and circles at Z_{FL}. The predicted upwash at Z_{FL}, using the "measured" upwash at Z_{SL} in Eq. (5) is shown using a dashed line. The coincidence between the measured data (circles) and the computation (dashes) indicates the absence of wall interference and confirms the validity of Eq. (5) in computing free-air upwash.

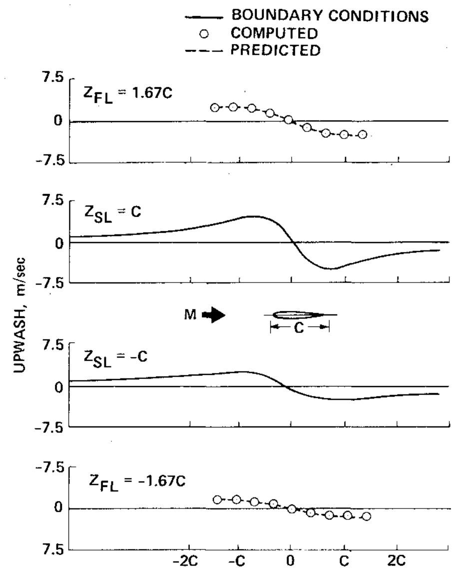

Fig. 2 Computational check of two-dimensional interference assessment method; NACA 0012 airfoil, $M = 0.65$, $\alpha = 2$ deg.

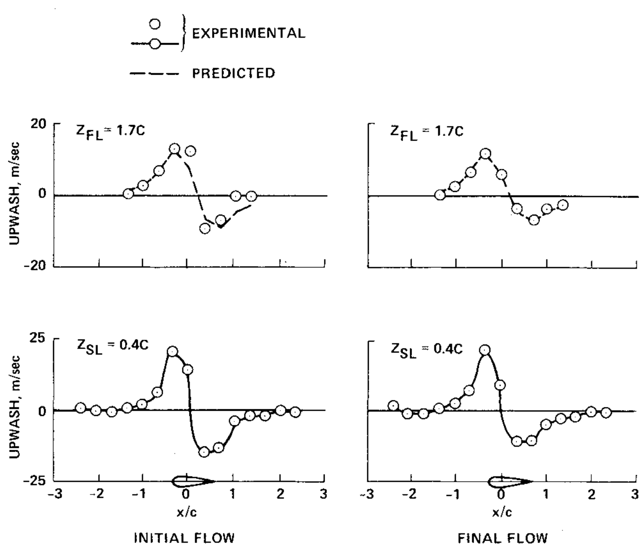

Fig. 3 Pilot wind tunnel test data showing convergence to interference-free flow; NACA 0012, $M = 0.78$, $\alpha = 0$ deg.

The two-dimensional scheme was further validated in a recently completed pilot wind tunnel test program. As reported in Ref. 18, the test was conducted in a 25×13 cm adaptive-wall wind tunnel using a 7.6 cm chord×25 cm span wing model having an NACA 0012 airfoil section. The adaptive-wall wind tunnel consisted of 20 separately controlled subplenums, 10 above and 10 below the model. Details concerning the test geometry, mechanical system, and iterative procedures for converging to interference-free flow conditions may be found in the aforementioned paper.

Measured upwash data obtained with a single-component laser anemometer were used in the interference assessment procedure. Typical results are presented in Fig. 3 (adapted from Ref. 18) where the measured and predicted upwash velocities are shown for the region above the airfoil at $M = 0.78$, $\alpha = 0$ deg. (Since the flow is symmetric, only the upper region is presented.) The discrepancy between theory and experiment at $z = Z_{FL}$ is obvious for the initial flow. By the third iteration, after the wall boundary conditions were modified appropriately, agreement was much better; it can be assumed that the final flow was interference free. The airfoil pressure distribution was validated by comparing it with other free-air data.[20] This comparison is shown in Fig. 4 where the

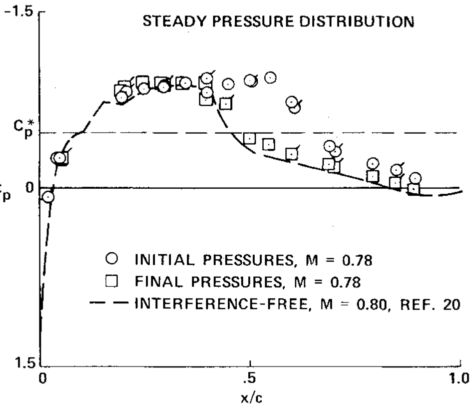

Fig. 4 Pressure data from pilot test showing convergence to free-air conditions; NACA 0012, $\alpha = 0$ deg.

pressure distributions for initial and final iterations at $M = 0.78$ are compared with the interference-free data from Ref. 20 that was closest to the measured conditions.

These pilot tests have validated the present scheme and have also indicated the sensitivity of the method to experimental error. The laser anemometer used to measure the upwash has an accuracy approaching 1 m/s; the errors in upwash due to wall interference are closer to 5 m/s. Therefore, present-day experimental practice and simple applications of the fundamental formula [Eq. (5)] are sufficient for practical implementation of the adaptive-wall concept for the class of flows investigated here.

IV. Three-Dimensional Flow

The geometry for the compatibility assessment problem in three dimensions is shown in Fig. 5. The freestream Mach number may be high enough to induce locally supercritical flow about the model, but not high enough to cause the sonic bubble to intersect the inner control surface Ω_{SL}. Given the measured upwash on all six faces of Ω_{SL}, the unconfined upwash is the solution to an exterior Dirichlet problem in three dimensions. In the region exterior to Ω_{SL}

$$\beta^2 \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} = 0 \tag{6}$$

subject to the boundary conditions

$$w = W_{SL} \text{ on } \Omega_{SL}$$

and

$$w \to 0 \text{ for } x^2 + y^2 + z^2 \to \infty$$

except in the wake where a trailing vortex model is included. The solution to the upwash equation is not the complete flow solution, but other velocity components can be found if desired.

Many techniques are available for solving this problem. For simple control surface shapes, it may even be possible to construct an analytical solution. The approach considered here is a numerical one that takes advantage of the planar control surfaces and the discrete nature of the boundary measurements. The main application is to support the adaptive-wall wind tunnel, and a key consideration in choosing a solution technique is that it be fast, reliable, and require minimum computer memory.

The choice of a numerical method and the constraints of a minicomputer environment makes an exact application of the far-field boundary condition difficult. Following the methods

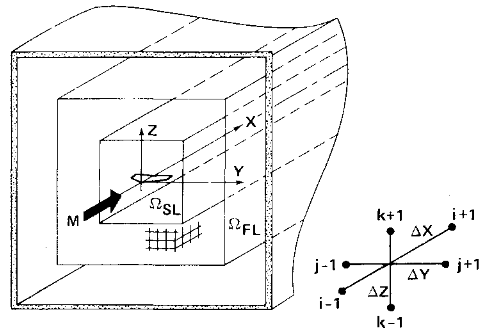

Fig. 5 Interference assessment in three dimensions.

commonly used in the numerical analysis of transonic flows,[21] a finite computational mesh is used and the upwash imposed on the outer boundary is the far-field representation of the flow induced by the lifting wing. The dominant term in the far field is the lift; it is modeled by a constant-strength horseshoe vortex with a total lift equivalent to the lift measured on the model. On the inner boundary Ω_{SL}, the measured upwash velocities at each mesh point are imposed. Finally, on the upstream and downstream faces of the computational network, the perturbation upwash is modeled by the same horseshoe vortex as that used for the outer boundary conditions.

The algorithm ultimately adopted is one that solves the centrally differenced second-order system of finite-difference equations for Laplace's equation [obtained from a Prandtl-Glauert scaling of the upwash equation, Eq. (6)] by using a direct solver at each cross-stream station and an iterative method in the streamwise direction. The mesh system and a typical computational molecule are shown in the inset in Fig. 5. The second-order central differencing of Laplace's equation has the following functional relationship among the seven mesh points:

$$w_{ijk} = \theta_1 w_{i+1,j,k} + \theta_2 w_{i-1,j,k} + \theta_3 w_{i,j+1,k} + \theta_4 w_{i,j-1,k} + \theta_5 w_{i,j,k+1} + \theta_6 w_{i,j,k-1} \tag{7}$$

where

$\theta_1 = \theta_2 = (A\Delta X^2)^{-1}$

$\theta_3 = \theta_4 = (A\Delta Y^2)^{-1}$

$\theta_5 = \theta_6 = (A\Delta Z^2)^{-1}$

$A = 2(1/\Delta X^2 + 1/\Delta Y^2 + 1/\Delta Z^2)$

w_{ijk} = upwash at the mesh point $i\Delta x$, $j\Delta y$, $k\Delta z$

Application of the algorithm at each mesh point reduces the partial-differential equation to a system of linear simultaneous algebraic equations. The system is solved iteratively in the streamwise and simultaneously in the lateral direction. If the unknowns at streamwise stations $(i-1)$ and $(i+1)$ in Eq. (7) are retained on the right side of the equation (along with the boundary conditions where appropriate) a system of equations is obtained whereby the unknowns consist only of the cross-stream upwashes at the current station i. Using the just-calculated solution at $(i-1)$ and a previous iteration at the $(i+1)$st station, values of w_{ijk} at all cross-stream stations are simultaneously computed using a variant of the classical $L-U$ decomposition scheme.[22] Solutions are obtained successively until the downstream boundary is reached whereupon successive iterations are computed as in the Gauss-Seidel scheme. The process is continued until successive iterations remain invariant within a given tolerance. This method was developed as a compromise

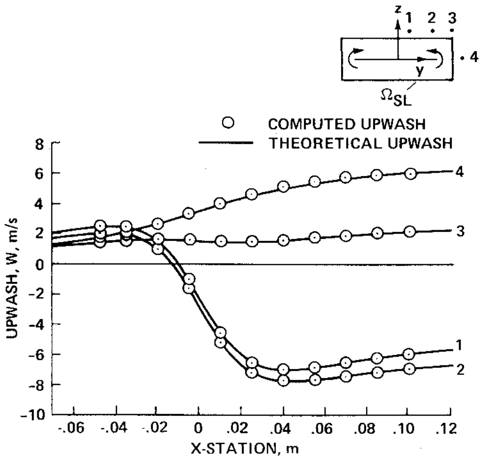

Fig. 6 Computational check of three-dimensional interference assessment method; horseshoe vortex in incompressible flow. Values of (Y, Z) in meters: point 1 (0.04, 0.045); point 2 (0.12, 0.045); point 3 (0.20, 0.045); point 4 (0.22, 0.0).

between a full simultaneous solution that was beyond the capabilities of the minicomputer and a simple point-iteration scheme that would take too long for on-line applications.

A sample calculation based on the method is presented in Fig. 6. The upwash induced by a horseshoe vortex with a semispan of 0.1697 m and a strength of 2.5 m^2/s was used as boundary conditions on Ω_{SL} and in the far field. The predicted upwash using the compatibility assessment method is compared with the analytical solution along four generators in Ω_{FL}. The good agreement between theory and calculation shows that the compatibility conditions are satisfied (as they should be) and the flow is interference free.

The outer flow was computed on a mesh with 11 stations in the X direction and 15×15 stations in the Y-Z crossflow plane. This computation took approximately 4 min, which is reasonable for pilot studies but will have to be reduced somewhat for practical applications. The computer used for the present computations was a Data General S/200 minicomputer with a speed of about 0.5 million floating point operations per second (MFLOPS). Given such specialized peripherals as array processors (with speeds of 6 MFLOPS) in conjunction with the new generation of minicomputer, it is expected that execution times will decrease significantly.

V. Concluding Remarks

A new compatibility assessment method for use in adaptive-wall wind tunnels has been described. The method is valid for both two- and three-dimensional flows and requires the measurement of only one velocity component. The method was developed for compressible subsonic flows at and beyond the control surface, but the basic method can be extended to fully transonic flow once adequate computational power is available.

The assessment procedure developed here is particularly adaptable to the use of a single-component laser anemometer, but any single-flow measurement at two surfaces (such as two pressure pipes) can be used as well.

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